Reducing sequence risk using trend following and the CAPE ratio

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Abstract

The risk of experiencing bad investment outcomes at the wrong time, or sequence risk, is a poorly understood, but crucial aspect of the risk faced by investors, in particular those in the decumulation phase of their savings journey, typically over the period of retirement financed by a defined contributions pension scheme. Using US equity return data from 1872-2014 we show how this risk can be significantly reduced by applying trend-following investment strategies. We also demonstrate that knowledge of a valuation ratio such as the CAPE ratio at the beginning of a decumulation period is useful for enhancing sustainable investment income.

Keywords: Sequence Risk; Perfect Withdrawal Rate; Decumulation; Trend Following; CAPE

JEL Classification: G10, G11, G22.

The topic of pension saving and decumulation is of growing importance in many parts of the world as companies retreat from defined benefit (DB) schemes leaving investment and withdrawal decisions to individuals. Some economists have focussed their attention on this important topic by proposing ever more creative accumulation and decumulation strategies. These strategies include frameworks for combining deferred annuities, state benefits, guaranteed annuity-type income, along with flexible income sourced from differing degrees of risky investment. However, these approaches are generally silent on the type of investment strategy needed for a successful accumulation and decumulation experience with risky assets. Instead, they have preferred to create risk-free benchmarks of index-linked bonds (see Sexaeur, Peskin and Cassidy (2012)). In our view, designing a savings and decumulation strategy without giving careful consideration to investment strategy is like designing all of the necessary elements of a car – chassis, gear box, braking system, etc – except the engine.

In this paper we want to shift the focus back to investment strategy – the risk engine. We make use of the concept of Perfect Withdrawal Rates (PWR) (Suarez et al, 2015) to investigate the decumulation experience since 1872 of a US investor with a 20-year investment horizon. We also explain and highlight the potentially pernicious impact that the sequence of investment returns can have when investors are withdrawing regular income from their investments. This is known as Sequence Risk. We find evidence to suggest that the application of a simple trend following filter to an equity investment can help generate returns with low drawdowns which, in turn, reduces sequence risk leading to enhanced PWRs. Another question that we address here is whether indicators of equity market valuation are useful for predicting the withdrawal rates at any point in time. In other words, does, say, a high cyclically-adjusted Shiller PE ratio (CAPE) suggest an overvalued market followed by equity price falls and a bad sequence of returns, leading to subsequent lower future PWRs? We find clear evidence to suggest that the CAPE can be used to help enhance withdrawal rates.

Withdrawal rates

The literature on optimal withdrawal rates in retirement can be traced back to Bengen (1994), where he presents the concept of “the 4% rule”. Bengen shows that a 4% withdrawal rate from a retirement fund, adjusted for inflation, is ‘usually’ sustainable for ‘normal’ retirement periods. Cooley, Hubbard, and Walz (1998, 1999, 2003, and 2011) then confirmed this conclusion, with similar findings using overlapping samples of historical stock and bond returns.

A crucial distinguishing feature of these papers is that they rely on a constant real withdrawal amount throughout the decumulation phase, with no ‘adaptive’ behaviour as circumstances change. A number of studies have introduced ‘adaptive’ rules: Guyton and Klinger (2006) manipulate the inflationary adjustment when return rates are too low, modifying the withdrawal amount, while Frank, Mitchell, and Blanchett (2011) use adjustment rules that depend on how much the rate of return deviates from the historical averages. Zolt (2013) similarly suggests curtailing the inflationary adjustment to the withdrawal amount in order to increase the portfolio’s survival rate where appropriate. Basically these withdrawal rates ‘adapt’ to changing circumstances.

An important extension of this research is to treat the planning horizon length as a stochastic variable (instead of fixed). The aim here (quite sensibly!) is to ensure that the funds in the retirement account “outlive” the retiree: Stout and Mitchell (2006) use mortality tables to make sure that the uncertain retirement period is considered, while Stout (2008) decreases the withdrawal amount whenever the account balance falls below a measure of the present value of the withdrawals yet to be made and increases it when the balance is above this measure. Mitchell (2011) similarly uses thresholds to initiate such adjustments.

A more theoretically coherent approach treats the selection of withdrawal amounts as a lifetime-utility maximization problem. Milevsky and Huang (2011) consider the total discounted value of the utility derived across the entire retirement period, where this length of retirement is a stochastic variable and the subjective discount rate is a given. Williams and Finke (2011) use a similar model with more realistic portfolio allocations.

Blanchett, Kowara, and Chen (2012) focus on a different concept. In their paper the researchers measure the relative efficiency of different withdrawal strategies by comparing the actual cash flows provided by each strategy to the flows that would have been feasible under perfect foresight, in other words they make use of the concept of a “perfect withdrawal rate”. This *Perfect Withdrawal Rate* (PWR) is the withdrawal rate that effectively exhausts wealth at death (or at the end of a fixed, known period) which can be identified if one had *perfect foresight* of all returns over that period. It can therefore be used as a benchmark for comparing competing investment strategies and for deriving a measure of a crucial risk faced by investors that are drawing income from investment portfolios: sequence risk. *Sequence Risk* is the risk of experiencing bad investment outcomes at the *wrong time.* Typically the *wrong time* is towards the *end* of the accumulation phase and at the *beginning* of the decumulation period, that is, it is symmetric around the time of retirement. Blanchett et al (2012) and Suarez et al (2015) construct a probability distribution for the PWR and use it to derive a new measure of *sequence risk* in the process. We use these ideas to show that a particular class of investment strategies (both simple and transparent) which tends to smooth returns, can offer superior Perfect

Withdrawal Rates across virtually the whole range of return' environments. It is this smoothing of returns that leads to a better decumulation experience across virtually all investing timeframes.

Calculating PWRs and deriving a measure of Sequence Risk

For any given series of annual returns there is one and only one constant withdrawal amount that will leave the desired final balance on the account after *n* years (the planning horizon). This is known as the Perfect Withdrawal Amount (PWA). It can also be expressed as a percentage relative to the initial value of the investment pot, in which case it is referred to as the Perfect Withdrawal Rate (PWR). The final balance could be a bequest or indeed could be zero. In the case of the latter, Suarez et al (2015) point out that identifying the PWR is equivalent to finding the fixed payment that will fully pay off a variable-rate loan after *n* years.

The basic relationship between account balances in consecutive periods is:

$$K_{i+1} = (K_i - w)\ (1+r_i) \tag{1}$$

where K_i is the balance at the beginning of year i, w is the yearly withdrawal amount, and r_i is the rate of return in year i in annual percent. Applying equation (1) chain-wise over the entire planning horizon (n years), we obtain the relation between the starting balance K_S (or K_1) and the end balance K_E (or K_n):

$$K_E = (\{[(K_S - w)\ (1+r_1) - w]\ (1+r_2) - w\}\ (1+r_3)\ \ldots - w)\ (1+r_n) \tag{2}$$

And we solve equation (2) for *w* to get:

$$w = [K_S \prod_{i=1}^{n}(1 + r_i) - K_E] / \sum_{i=1}^{n} \prod_{j=i}^{n}(1 + r_j) \tag{3}$$

Equation (3) provides the constant amount that will draw the account down to the desired final balance if the investment account provides, for example, a 5% return in the first year, 3% in the second year, minus 6% in the third year, etc., or any other particular sequence of annual returns. This is the Perfect Withdrawal Amount (PWA).

Quite simply, if one knew in advance the sequence of returns that would come up in the planning horizon, one would compute the PWA, withdraw that amount each year, and reach the desired final balance exactly and just in time.

Numerous studies provide examples of a sequence of, say, 30 years of returns generated possibly with reference to an historical period or via Monte Carlo simulations, and offer the unique solution of the PWA. It involves withdrawing the same amount every year, giving the desired final balance with no variation in the income stream, no failure and no surplus. As we noted above, Blanchett et al (2012) present a measure similar to PWA called the Sustainable Spending Rate (SSR). Suarez et al (2015) point out that the PWA is a generalization of SSR, with SSR being the PWA when the starting balance is $1 and the desired final balance is zero.

So every sequence of returns is characterised by a particular PWA, or PWR and hence the retirement withdrawal question is really a matter of "guessing" what the PWA will turn out to be (eventually) for each retiree's portfolio and objectives. So the problem now becomes how to estimate the probability distribution of PWAs from the probability distribution of the returns on the assets held in the retirement account.

Note that the analysis so far offers a number of useful insights into sequence risk measurement. First, Equation (3) can be restated in a particularly useful way since the term $\prod_{i=1}^{n}(1+r_i)$ in the numerator is simply the cumulative return over the entire retirement period, (call it *Rn*).

The *denominator*, in turn, can be interpreted as a measure of sequencing risk:

$\sum_{i=1}^{n} \prod_{j=i}^{n} (1 + r_i)$ = (1+*r1*)(1+*r2*)(1+*r3*)…(1+*rn*) + (1+*r2*)(1+r*3*)…(1+*rn*) + (1+*r3*)(1+*r4*)…(1+*rn*) + …. + (1+*rn-1*)(1+*rn*) + (1+rn) (4)

The interpretation of this is straightforward: for any given set of returns equation (4) is *smaller* if the *larger* returns occur *early* in the retirement period and *lower* rates occur at the *end*. This is because the later rates appear more often in the expression. Suarez et al (2015) suggest the use of the reciprocal of equation (4) to capture the effect of sequencing: so let $S_n = 1/\sum_{i=1}^{n} \prod_{j=i}^{n} (1 + r_i)$. This falls as the sequence becomes more favourable, and even though one set of returns appearing in 2 different orders will have the same total return (i.e. R_n with different S_n values), so the PWA rates will be different.

Normally the financial analysis of investment returns focus on *total* return and some reward to risk measure such as the Sharpe ratio where there is no consideration of the return sequence. But in both accumulation and decumulation the *order* of returns matters. An example will make this clearer. Consider the three sets of returns presented in Table 1; clearly the mean, volatility and Sharpe (and indeed Maximum Drawdown) are the same in each case, but the returns' sequence differ as is evidenced by the different values of Sequence Risk ($1/S_n$) with lower values of this metric associated with higher PWRs.

This allows a useful, highly intuitive simplification of equation (3) in the form of equation (5), such that the PWA depends positively on total return, R_n, the starting amount, K_s, and the measure of sequence risk, S_n, and negatively on the final amount, K_E:

$$w = (R_n K_s - K_E)\, S_n \qquad (5)$$

Table 1 and equation 5 make it clear that it is not simply the total return that matters but the order in which the component returns occur: if 'good' returns come early in the sequence then the PWA will be larger than if they occur later.

Other studies have tried to account for sequence risk (Frank and Blanchett, 2010; Frank, Mitchell, & Blanchett, 2011; Pfau, 2014), often developing proxy variables to measure the correction required due to the sequencing issue. Suarez et al (2015) suggest that equation (5) comes directly from the simplest, most natural interpretation of the problem, that is that S_n is not a proxy but a measure of what they term '*orientation'* (return rates going up, going down, up a little then down a lot, etc.), and this is the crucial concept for assessing sequencing.

Finally we should note that *w* (the PWA) can be transformed into a withdrawal *rate* by dividing equation (5) by K_s

$$w/K_s = R_n S_n - S_n(K_E / K_S) \qquad (6)$$

Note that if we have a bequest motive then we simply need to know the fraction of the initial sum to be bequeathed to calculate the PWR. As Suarez et al (2015) point out, in contrast to simplistic financial planning solutions, to set aside a bequest sum beforehand is not necessary as these funds can also generate returns and be used for consumption. Setting aside a sum is simply a special case of the above general form expressed in equation (6).

In this paper we make use of the concept of PWRs to compare investment strategies over a 20 year decumulation horizon. But of course, not everyone will live for twenty years in retirement. First, it is important to point out that the analysis that we conduct can be adapted for any withdrawal horizon. But second, and more importantly, we are not suggesting that the strategies that we examine should be the only source of income for retirees. Instead we believe that the investment strategies that we investigate should be combined with, say, a deferred annuity that kicks in at the end of the chosen decumulation period (see for example Merton, 2014 and Sexauer et al, 2012, for a fuller discussion of the potential benefits of this dual approach to funding one's retirement).

Constructing a Probability Distribution for PWRs for an all equity portfolio

Much of the financial planning literature aims to make probability statements regarding the chance of running out of funds given any particular withdrawal rate and planning horizon. So we now create a probability distribution for the PWR/PWA using a long-run of monthly equity returns extracted from the Shiller website[1]. This all-equity portfolio may be considered rather unlikely as an investment choice in practice, but it serves to illustrate our key points regarding the choice of investment strategy. In practice of course investors may wish to hold a proportion in bonds and other asset classes to benefit from diversification and to align the investment portfolio's risk to levels of risk tolerance. A surprising result may well be that a 100% equity portfolio is not such a bad idea, *providing* that one overlays it with trend-following filter.

Assuming that we have perfect foresight, what would the real PWR look like through time assuming a 20 year decumulation period? This is shown in Figure 1 where, as throughout this paper, we assume a zero bequest intention. We focus here on the blue line which shows the PWR generally varying between 8% and 12%, but occasionally straying as low as 4% in 1930 and as high as 15% in 1949. For several years in the 1980s it is well above 10%. This suggests two things. First, there is a huge variation in the ability to withdraw cash from a retirement pot

[1] http://www.econ.yale.edu/~shiller/data.htm

depending on the accident of one's birth date. Second, *all* of the rates are above 4%, giving very long term succour to Bengen's (1994) 4% rule (at least over 20 year periods).

Now we know what the history of PWRs would look like with perfect foresight for the 100% S&P500 portfolio, we can construct a probability distribution for this particular investment strategy. We begin with 100% invested in this equity portfolio. We calculate the real returns on the S&P500 for each year over the period 1872 to 2014. We then use Monte Carlo techniques to draw 20 years of return, one at a time, with replacement. These sets of return are then interpreted as the real returns over a 20-year investment horizon, in the order in which they were drawn, that an investor might experience. This process is repeated 20,000 times, allowing us to compute the cumulative return (R_n) and sequencing factor (S_n) for each series of returns obtained. This provides us with 20,000 (R_n, S_n) pairs. The blue line in Figure 2 presents the frequency distribution of the PWA formula (equation (5)) evaluated at each of these 20,000 (R_n, S_n) pairs, using $100,000 as a starting balance and with $0 as a desired terminal balance. The second column in Panel A of Table 2 contains the distribution's percentiles. Taken together these results are broadly comparable with Figures 3 and 4 in Suarez et al (2015), albeit with real PWRs and a 20 year investment horizon. We can interpret the distribution of PWRs as follows: there is a 1% chance of a real PWR of 2.95% or less; a 10% chance of a real PWR of 5.01% or less; and a 50% chance of a real PWR of 8.64% or more. Given that the final amount is $0, any overshoot in withdrawing results in ruin. Hence we could say that 50% of the Monte Carlo withdrawal runs produced real PWRs less than 8.64% so that failure risk for withdrawing over 8.64% is 50%. Similarly failure risk for withdrawing over 5% p.a. is about 10% (i.e. 10% of runs produced real PWRs of over 5.01%).

The inverse of failure risk is surplus risk and this can be estimated by inverting the roles of PWR and the end balance. For a given end balance and PWR we can say that a surplus accrues a certain percentage of time reflecting the occurrence of PWRs greater than that chosen. In fact, in the Suarez et al example, with a nominal perfect withdrawal amount of $43,000 p.a. (i.e. a 4.3% withdrawal rule in their case), 74% of the Monte Carlo runs end up with more money than they began with; in 58% of the runs the final balance was double the starting balance; and it would have a 12% probability of ending up with 10 times the initial sum.

Finally, in column 2, Panel B of the table we present some descriptive statistics for the real, buy-and-hold, annual returns on the S&P500 over this period, that is, the descriptive statistics of the risk engine. Note in particular the very high maximum drawdown of 76.8%.

Trend Following and Sequence Risk

Clearly from equations (6) and (7) the sequence risk measure, S_n, influences the PWR directly: equation (6) shows that the more favourable sequencing, S_n, gives a higher PWR. More favourable sequencing is associated with relatively good returns early in the planning period (see Okusanya (2015)). In particular, the avoidance of heavy losses in the early phases of decumulation is crucial for high PWAs. But if asset returns are unpredictable, how can we secure a favourable S_n? Milevsky and Posner (2014) investigate how and when traded equity options can help reduce sequence risk and, in so doing, be used to extend the life of a retiree's investments. However, another very straightforward solution is to acknowledge that while the *order* of returns cannot be predicted, it may be possible to produce investment strategies that offer substantially reduced return volatility or, more precisely, much reduced drawdown in returns since reduced volatility in itself is not enough to secure a high PWA. Indeed, while there is no precise mathematical relationship between maximum drawdown and sequence risk we suggest that a low maximum drawdown should be associated in practice with more favourable sequence outcomes.

While diversifying across asset classes should nudge portfolio returns in the desired direction with improved risk-return, and possibly lower maximum loss experiences, there is an even more powerful technique which can be applied to individual asset classes with dramatic effect: this is *'trend-following'*, where one invests in an asset when it is in an uptrend (defined as a current value above some measure of recent past average) and switch into cash when the current value is below such an average[2]. This approach to investing has been explored by a number of researchers in the past. Faber (2007) shows how this simple approach can be applied across a range of broad US asset classes as a disciplined way of implementing asset allocation decisions to produce multi-asset class portfolios with higher returns and lower volatility[3]. ap Gwilym et al (2010) also find that trend following filters can enhance risk-adjusted returns. For example, they show how the approach can halve the maximum drawdown on an investment in the MSCI World index, compared to a simple buy-and-hold strategy over the period 1971 to 2008. Hurst et al (2012) expand the universe of asset classes considerably in their research into the properties of trend following filters. They apply the trend following filter to 24 commodities markets, 11 equity markets, 15 bond markets and 9 currency pairs, using data from 1903 to 2012. They find that the approach "*delivered strong returns and realized a low correlation with traditional asset classes*" over that period. Clare et al (2013) find similar evidence when applied to the S&P500, and essentially conclude that the simple ten month moving average signal[4], also applied in this paper, produces superior risk-adjusted returns than more complex

[2] There is a tendency in the finance industry to use the terms 'momentum' and 'trend following' almost interchangeably, and yet they are subtly different. Trend following, although closely related to momentum investing, originally identified by Jegadeesh and Titman (1993), is fundamentally different in that it does not order the past performance of the assets of interest, though it does rely on a continuation of, or persistence in, price behaviour based on some technical rule. Moskowitz et al (2012) refer to the trend following filter that we use here as "time series momentum", and refer to the Jegadeesh and Titman momentum effects as "cross-sectional momentum".

[3] See also Clare et al (2016) for evidence of how trend following filters can be used to enhance asset allocation by predominantly reducing maximum drawdowns on portfolios.

[4] The authors also show that the ten month calculation period for the average is not critical for their results. They show that 6, 8, 10 and 12 month calculation rules produce very similar results. When examining the usefulness

technical rules, such as those relating to cross-over points, and that any positive return enhancement applying such trend following rules at a daily frequency is nearly always offset by higher transactions costs.

Our basic hypothesis then is that applying a simple, monthly trend-following rule to any series of asset returns dampens volatility, typically maintains or increases returns over longer periods, and substantially reduces maximum drawdown for that series, which should in turn lead to lower sequence risk[5]. To test this hypothesis we now replace the buy-and-hold equity investment strategy with an equity strategy that incorporates a trend following filter. To this end, we create a set of trend following returns by using a ten month moving average of the S&P500. At the end of each month if the index value is greater than its ten month moving average the investor earns the return on the S&P500 in the subsequent month. However, if at the end of the month the index value is below its ten month moving average the investor, switches into cash and earns the return on cash in the subsequent month. The trend following rule therefore switches the investor between equities and cash depending upon the level of the index relative to its ten month moving average.

The brown line in Figure 1 presents the real PWR through time assuming a 20 year decumulation period, and assuming perfect foresight. We can see from the chart that it is generally higher than the equivalent series generated by the equity buy-and-hold risk engine

of the same trend following rule, but for a range of asset classes, Clare et al (2016) also find that the results are not sensitive to the choice of the moving average calculation.

[5] For trend following filters such as the one used in this paper to produce attractive, risk-adjusted returns by reducing maximum drawdowns, broad markets need to "trend" over periods greater than the frequency of the application of the rule. The findings of Faber (2007), ap Gwilym et al (2010), Hurst et al (2012), Clare et al (2016) (and others) using a range of asset classes and sample periods, suggest that financial markets do tend to trend. The trending in markets is, in turn, probably related to behavioural drivers such as herding, overconfidence, etc. If true, then for as long as these biases exist, markets will tend to trend, and trend following will continue to produce attractive risk-adjusted returns. However, the question of why trend following works is really beyond the scope of this paper. We use it here to demonstrate that it is possible to implement a strategy that could bring investors closer to their Perfect Withdrawal Rate – others may know of, or prefer other investment strategies.

[the blue line], which is an encouraging start. We now calculate the distribution of the PWR generated by the trend following equity risk engine, by first calculating the real return achieved in each calendar year of our sample from applying the trend following filter. We then repeat the Monte Carlo analysis, but this time drawing from the set of real annual returns generated by the trend following rule[6]. The brown line in Figure 2 shows the distribution of the PWRs generated by the trend following investment strategy. There is a substantial shift to the right in the distribution compared with the distribution produced by the buy-and-hold equity strategy (represented by the blue line in Figure 2) and it is much more concentrated around its median value of circa 9%. The final column in Panel A of Table 2 presents the percentiles of this distribution and shows that around 90% of the time the PWRs produced by the trend following risk engine are greater than produced by the equivalent buy-and-hold equity strategy (shown in the second column in Panel A). In fact at lower probability levels the PWRs are nearly double those for the 100% buy-and-hold equity strategy.

The final column in Panel B of Table 2 provides summary statistics of the annual real returns generated by the trend following strategy and gives a clue to the superior PWRs achieved by using the trend following strategy. The average real return of 8.84% produced by the trend following strategy compares very favourably with the 6.82% produced by the buy-and-hold strategy. However, perhaps even more important is the one-third reduction in volatility from 14.29%pa to 9.86%pa, and the halving of maximum drawdown from 76.8% to 34.88%. In keeping with the findings of previous research in this area, the trend-following filter applied here reduces both volatility and maximum loss. This in turn leads to a reduction in sequence

[6] An alternative to the Monte Carlo approach described here would have been to analyse instead the 1,500 unique twenty-year periods. Following the suggestion of anonymous referee we conducted the analysis of these 1500 series. We found that the average PWR for the buy-and-hold and trend following strategies were 8.8% and 9.6%, a difference that we found to be statistically significant at 99% level of confidence using standard errors robust to moving average errors. These results, the empirical distribution and the data are available on request from the authors.

risk, allowing for noticeably higher PWRs in virtually all cases except those greater than the 90th percentile.

But what about transactions costs?

In achieving the lower sequence risk the trend following filter requires that the investment is switched between the risky asset class, in this case the S&P500, and cash. Before dealing with the knotty issue of historical transactions costs however, we can first say something about holding costs. The trend following literature discussed above has found for a wide range of asset classes and historic investment periods that the trend following filter described here tends to require investment in the risky asset class for about two thirds of the time, indeed, this is also our finding for the S&P500 for the period 1872 to 2014. On the assumption that a cash balance attracts a much lower holding fee than an investment in equities, over the long term an investor in a trend following portfolio should expect to pay only two thirds of the holding costs (management fee) that they would otherwise pay as a result of a buy-and-hold strategy in the same risky asset.

A transactions charge would only be payable in the event of a switch. ap Gwylym et al (2010) apply the trend following filter used here to the MSCI World index, in dollars, from 1971 to 2008 and report 7 switches (or round trip trades) per decade. We find that the number of switches for the S&P500 for the nearly 150 year period analysed here was also just under 7 per decade. With regard to switching costs, Hurst et al (2012) investigate the benefits of trend following applying the sort of trend following filter used in this paper to a century of US capital market data, finding that the filter enhances risk-adjusted returns considerably over the last century or so. In arriving at these results they estimated, and used one way transactions costs as a proportion of the value of the investment for developed economy equities as being 0.36%, 0.12% and 0.06% for the periods 1903 to 1992, 1993 to 2002 and from 2003 to 2012 respectively. Finally, in their investigation of trend following using data from 1994 to 2015, for a range of asset classes, Clare et al (2016) base equity transactions costs on ETF fees, using a one way transactions cost of 0.20% of the value of the investment and find that the trend following filters still outperform buy-and-hold comparable investments by an impressive margin. For example, they find that applying the same, simple 10 month moving average signal used in this paper that the approach applied to developed economy equities, produced an annualised return of 8.0%, a Sharpe ratio of 0.79 and a maximum drawdown of 11.6%; whereas the equivalent buy-and-hold portfolio produced an annualised return of 6.6%, a Sharpe ratio of 0.33 and maximum drawdown of 46.6%.

To add further insight into the possible impact of transactions costs we calculated a "break-even" switching fee, which we define as: the one-way transaction cost that would equate the returns on a Trend Following strategy applied to the S&P500 with those produced by a 'buy-and hold' investment in the S&P500, over the full sample period. This break-even value turned out to be 1.35%. To put this into perspective, Hurst (et al) suggest that a one-way transaction cost of 0.36% should be used for US equities between 1903 and 1992; while Jones (2002) estimates one-way transactions costs as averaging 0.38% on US stocks between 1900 and 2001. We recalculated the 20-year Perfect Withdrawal Rates generated by the trend following strategy applied to the S&P 500. When we set the one-way transaction cost to Hurst et al's recommendation of 0.36% we find that the average PWR is 9.68%, compared with the S&P500 Buy and Hold approach which produces a PWR of 8.8%. Finally, since no investors will be implementing a strategy like this in the past, it is probably pertinent to consider Hurst et al's estimate, based on their practical experience, of the one-way transaction costs for trading US equities between 2003 and 2012, which they report as 0.06%.

So holding costs of trend following strategies are generally two thirds that of a buy-and-hold equivalent and switches are relatively rare, at least when trend following is applied at the lower, monthly frequency, and were unlikely to have been high enough to eliminate the benefits of the trend following approach in the past.

Can Equity Valuation Measures help in securing higher withdrawals?

The relationship between CAPE and PWRs

If a simple trend-following investment strategy facilitates superior withdrawal rates most of the time, it is natural to ask whether other market timing or valuation indicators can help identify withdrawal amounts to give similarly "improved" solutions. In particular, measures such the CAPE ratio (Shiller, 2001) have been shown to have some predictive power for longer-run equity returns[7]. Figure 3 shows the time-series plot of beginning-period CAPE (right-hand axis) against the 20-year real PWR generated using by the buy-and-hold equity strategy. If the earnings' yield is high (and hence CAPE is low) it is possibly indicative of above average equity returns and hence we would expect a higher perfect foresight PWR for the subsequent 20-year period. This is seen clearly in Figure 3, with low points for the CAPE in 1920, 1930 and 1980 being associated with high, subsequent PWRs.

We now move on to examine two very different periods of financial market history[8], the 20 year period from 1973 and the 20 year period from 1995, to examine in more detail: first, the potential benefits of updating withdrawal rates on annual basis using our Monte Carlo method; second, to examine the benefits of trend following in this adaptive PWR framework; and finally

[7] Blanchett et al (2012) introduce both bond yields and the CAPE ratio as indicators of market valuation.

[8] A common feature of the financial planning literature is the analysis of different periods of financial history to explore sustainable withdrawal rates in very different environments (for example see Chatterjee et al, 2011).

to investigate the possibility of integrating the “predictive” qualities of the CAPE, also updated annualy.

The period 1995 to 2015

Table 3 presents the results for the 20-year period beginning in 1995 where we use the buy-and-hold equity portfolio as the risk engine. The second column in the table shows that real equity returns for the first 5 years of the sample were very high indeed, suggesting the likelihood of low sequencing risk; this is indeed the case and the perfect foresight, real PWR is 10.781%, giving a real PWA of $10,781 p.a. for each of the 20 years.

In the columns headed “Monte Carlo Median PWA” in Table 3, we have generated a new, median PWA for each year in our sample. We do this by applying the Monte Carlo process described earlier, but on an annual basis to generate the PWA median. More precisely, we begin by generating the PWR using the Monte Carlo process and 20,000 annual, real return draws, we then calculate the median of the generated distribution to give the PWA in the first year (which is $8,545). At the end of the first year we repeat this exercise, but now the investment horizon is 19, rather than twenty years, so we now draw a series of 19 annual real returns 20,000 times to create a new distribution and median PWA, and so on. The median PWA will change each year depending upon the value of the investment pot at the end of the previous year[9]. Using this process we can see in Table 3 that after the initial 5 years’ of good investment performance the investment pot reaches over $188,000 with 16 years to go, allowing for a withdrawal amount of $19,654. Things then take a turn for the worse in 2008

[9] We believe that this is a simpler approach to creating adaptive PWAs than the adaptive rules found in the previous literature, for example, see Bernard (2011), Blanchett and Frank (2009), Guyton (2004), Mitchell (2011), Pye (2000), Robinson (2007) and/or Stout and Mitchell (2006).

where a 39% fall in the S&P leads to a fall in the PWA from $10,629 to just under $6,000 for 2009 (with 6 years remaining).

The final set of results presented in the columns headed “CAPE-based PWA” are generated by using the fitted value of the PWR based upon the CAPE at the beginning of each year and the simple linear regression described above, updated annually to the end of each prior year. The inverted CAPE values are given in column 3 of the Table (headed “EY”). The fairly low withdrawal rates in the early years, together with robust investment returns, leads to wealth reaching over $216,000 by the end of 1999. Together with the CAPE-driven PWRs, this leads to higher withdrawal amounts in the final years than those suggested by the Monte Carlo method. For example, the final withdrawal without the CAPE information is $8,715, compared with $14,037 with it. It would therefore appear then that knowing the value of the CAPE at the start of the year, could lead to a superior withdrawal experience for investors.

In Table 4 we repeat all of the calculations presented in Table 3 but where the risk engine is now the trend-adjusted S&P500 real returns. First, we note that the perfect foresight PWR of 12.31% is higher than the 10.78% presented in Table 3. So using trend following filtered returns leads to a higher PWR, as we have seen previously. Also note how the real returns generated by the trend-adjusted strategy presented in column 2 of the table, leads to a real return of -4.4% in 2002 compared to the -22% generated by the buy-and-hold strategy and reported in column 2 of Table 3, while a real return of 1.3% in 2008 compares favourably to the -39% generated by the buy-and-hold strategy in the same year. The higher real returns in these two years in particular facilitate higher PWAs. For example, the withdrawal in 2014 is $10,991 based on the trend following approach compared to $8,715 using the buy-and-hold risk engine. However, an even higher withdrawal rate is achieved when we combine trend following returns with information from the CAPE. For example the last three withdrawals are $12,847, $13,188 and $15,868 which compare very favourably with the Monte Carlo results produced by the unadjusted raw equity returns in Table 3 of $6,941, $7,410 and $8,715 respectively.

It would seem then that the trend following approach, combined with the predictive power of the CAPE together have the potential to produce a much better retirement experience in a period when raw investment returns are high in the early years.

The period 1973 to 1993

What happens if we now repeat this exercise over a period of financial history characterised by poor returns in the early years, for example the 20 years beginning 1973?

The second column in Table 5 shows real US equity returns for each year from 1973 to 1992. In 1973 and 1974 real returns were -23% and -34% respectively, suggesting the possibility of high sequencing risk for anyone starting the decumulation journey in 1973. Although returns recovered later in the period the damage was done: the perfect foresight PWR, shown in the table, was only 4.59% for the 20 year period, emphasising that accidents of birth date can have a major bearing on one's income in retirement. Both the median Monte Carlo and CAPE valuation metric produce substantially reduced PWAs relative to those reported in Table 3. For example, Table 3 shows that the Monte Carlo median approach gives a final withdrawal amount of $8,715, while the CAPE-based approach yields a final withdrawal value of $14,037; the equivalent values, shown in Table 5 for the 1973 to 1992 period, are $5,664 and $4,812 respectively.

But what if we now use trend-adjusted equity returns and repeat this exercise over this historical period? Table 6 contains these results for the trend-adjusted equity returns. First of all note the absence of really severe negative returns in column 2, which allows the perfect foresight PWR to rise by a third to 6.148% pa. Similarly the Monte Carlo and CAPE-based results suggest much higher withdrawals are possible, particularly in the early years, relative to the trend-unadjusted returns reported in Table 5. However, Table 6 shows that the CAPE-based annual withdrawals are not as high as those produced by the Monte Carlo approach. In this case then trend following alone produces the best withdrawal results.

Conclusions

In this paper we have drawn attention to a number of key features of the much neglected investment aspects of retirement planning and execution. We have also seen how accidents of birth date can dramatically impact retirement income. While the reduction of sequence risk may be recognised by financial planning professionals as an important aspect of the decumulation journey, there is relatively little awareness of it in the mainstream asset management and investing strategy literature, possibly because there is no widely accepted measure of it in practice. The challenge of creating investing strategies for the decumulation phase beyond the risk-free TIPS portfolios of, say, Sexaeur et al (2012), has barely begun: the choice would seem to be between controlling tail-risk with derivatives (Milevsky and Posner, 2014), versus portfolio timing adjustments into and out of cash (Strub, 2013). This study is firmly in the latter camp. We show that employing a simple trend following strategy results in significantly reduced sequence risk while generating a robust level of average returns and therefore an enhanced feasible withdrawal rate. We also show that there is potentially useful information in market valuation measures, such as the CAPE ratio, which might help guide withdrawal rates when we adapt these on a regular, possibly, annual basis.

The analysis in this paper represents our attempt at identifying a possible risk engine that may bring investors closer to their own perfect withdrawal rate. However, we acknowledge that there may well be alternative, superior investment strategies out there. And certainly a process that encompasses multiple asset classes, rather than just the equity-only approach investigated here, may provide even better defence against the pernicious effects of sequence risk. We therefore believe that the research focus should shift to the identification of suitable risk engines for decumulation journeys. Others have already designed a fine chassis, but a chassis without an engine will simply rust in the back yard.

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Table 1
Example of Sequence Risk

In this table we show the impact on Sequence risk ($1/S_n$) and the Perfect Withdrawal Rate (PWR) of three series of returns which have the same arithmetic mean (Mean), standard deviation (St. Dev.) and maximum drawdown (Max Draw).

Year	Return set 1	Return set 2	Return set 3
1	20%	-20%	0%
2	10%	-10%	10%
3	0%	0%	-10%
4	-10%	10%	-20%
5	-20%	20%	20%
Mean	0.00%	0.00%	0.00%
St. Dev.	15.8%	15.8%	15.8%
Max Draw	-20.0%	-20.0%	-20.0%
$1/S_n$	3.98	5.98	4.92
PWR	23.87%	15.90%	19.30%

Table 2
Real Perfect Withdrawal Rate Percentiles as a Percentage of Initial Balance

In Panel A of this table, in the column headed "S&P", we present the percentiles of Perfect Withdrawal Rates (PWRs) for an investor with a twenty year decumulation horizon, a starting investment balance of $100,000 and a desired investment balance of $0, based upon the total returns generated on a buy-and-hold investment in the S&P 500 from 1872 to 2014. Analogous results are presented in the column headed "S&P with trend following", where the returns have been generated by applying a trend following rule to real S&P returns as described in the text from 1872 to 2014. The distribution on which the percentiles were derived, were generated by Monte Carlo techniques which involved drawing 20 years of 12 monthly return values at random with replacement 20,000 times. Panel B of this Table presents the descriptive statistics of a buy-and-hold investment in the S&P (column headed "S&P") and for an investment in the S&P where returns have been generated by applying a trend following rule (column headed "S&P with trend following").

	S&P	**S&P with trend following**
	Panel A	
Percentile	%	%
1	2.95	5.57
5	4.20	6.61
10	5.01	7.21
20	6.11	8.03
30	7.00	8.67
40	7.85	9.23
50	8.64	9.80
60	9.43	10.38
70	10.38	11.01
80	11.47	11.81
90	13.05	13.00
95	14.37	14.04
99	16.87	16.22
	Panel B	
Annualized Real Return (%)	6.82	8.84
Annualized Real Volatility (%)	14.29	9.86
Maximum Real Drawdown (%)	76.8	34.88

Table 3

20-Year Decumulation Starting in 1995, based on buy-and-hold S&P 500 real returns

In this Table we present statistics for an investor beginning a 20-year decumulation period beginning in 1995, where investment returns are all driven from a buy-and-hold investment in the S&P500. The second column in the table presents the annual, real return achieved from investing in a buy-and-hold S&P 500 equity portfolio. The third column in the table presents the 1/CAPE value (EY) at the start of each decumulation year. The columns under the heading "Perfect Foresight PWA", present the value of the investment fund at the start of each year, the annual, perfect foresight withdrawal amount, and the value of the investment fund at the end of each year respectively. The columns under the heading "Monte Carlo Median PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the Median PWA has been determined by the Monte Carlo technique described in the text which is applied using data up to the start of the next withdrawal year. The columns under the heading "CAPE-Based PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the PWA has been determined at the start of each year by the CAPE regression described in the text.

			Perfect Foresight PWA			Monte Carlo Median PWA				CAPE-Based PWA			
Start year	Real Ret (%)	EY Start	Value start ($)	Withdrawal ($)	Value end ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
1995	35.0	5.02	100,000	10,781	120,439	100,000	8.55	8,545	123,458	100,000	6.91	6,910	125,666
1996	19.6	4.00	120,439	10,781	131,140	123,458	8.92	11,013	134,473	125,666	6.55	8,233	140,437
1997	29.6	3.61	131,140	10,781	155,943	134,473	9.18	12,340	158,242	140,437	6.59	9,256	169,966
1998	23.5	3.03	155,943	10,781	179,278	158,242	9.58	15,153	176,717	169,966	6.55	11,140	196,154
1999	18.4	2.58	179,278	10,781	199,446	176,717	9.99	17,647	188,287	196,154	6.60	12,956	216,848
2000	-8.8	2.26	199,446	10,781	171,980	188,287	10.44	19,654	153,721	216,848	6.79	14,734	184,241
2001	-14.2	2.68	171,980	10,781	138,385	153,721	10.82	16,625	117,693	184,241	7.51	13,828	146,295
2002	-22.0	3.28	138,385	10,781	99,480	117,693	11.19	13,167	81,489	146,295	8.43	12,333	104,437
2003	20.0	4.33	99,480	10,781	106,445	81,489	11.63	9,476	86,421	104,437	9.71	10,143	113,160
2004	9.3	3.76	106,445	10,781	104,522	86,421	12.46	10,772	82,653	113,160	10.24	11,582	110,983
2005	3.5	3.68	104,522	10,781	97,067	82,653	13.22	10,929	74,269	110,983	11.27	12,504	101,974
2006	11.4	3.78	97,067	10,781	96,153	74,269	14.34	10,652	70,892	101,974	12.55	12,797	99,375
2007	2.1	3.67	96,153	10,781	87,203	70,892	15.70	11,131	61,043	99,375	13.84	13,753	87,458
2008	-39.3	3.85	87,203	10,781	46,397	61,043	17.41	10,629	30,607	87,458	15.53	13,582	44,851
2009	26.6	6.51	46,397	10,781	45,084	30,607	19.57	5,989	31,162	44,851	19.11	8,572	45,924
2010	12.3	4.92	45,084	10,781	38,534	31,162	22.83	7,115	27,014	45,924	21.61	9,922	40,443
2011	-0.8	4.47	38,534	10,781	27,521	27,014	27.82	7,515	19,337	40,443	26.20	10,595	29,600
2012	14.8	4.87	27,521	10,781	19,218	19,337	35.90	6,941	14,230	29,600	34.41	10,187	22,286
2013	27.8	4.71	19,218	10,781	10,781	14,230	52.07	7,410	8,715	22,286	50.71	11,302	14,037
2014	15.0	4.02	10,781	10,781	0	8,715	100.00	8,715	0	14,037	100.00	14,037	0

Table 4

20-Year Decumulation Starting in 1995, based on real S&P 500 returns with trend following overlay

In this Table we present statistics for an investor beginning a 20-year decumulation period beginning in 1995, where investment returns are all driven by the real return on the S&P500 with a trend following overlay. The second column in the table presents the annual, real return achieved from investing in a buy-and-hold S&P 500 equity portfolio. The third column in the table presents the 1/CAPE value (EY) at the start of each decumulation year. The columns under the heading "Perfect Foresight PWA", present the value of the investment fund at the start of each year, the annual, perfect foresight withdrawal amount, and the value of the investment fund at the end of each year respectively. The columns under the heading "Monte Carlo Median PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the Median PWA has been determined by the Monte Carlo technique described in the text which is applied using data up to the start of the next withdrawal year. The columns under the heading "CAPE-Based PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the PWA has been determined at the start of each year by the CAPE regression described in the text.

			Perfect Foresight PWA			Monte Carlo Median PWA				CAPE-Based PWA			
Year	Real Ret (%)	EY Start	Start ($)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
1995	32.4	5.02	100,000	12,308	116,082	100,000	9.75	9,746	119,473	100,000	8.21	8,213	121,503
1996	19.6	4.00	116,082	12,308	124,104	119,473	10.03	11,988	128,541	121,503	7.85	9,534	133,903
1997	29.6	3.61	124,104	12,308	144,850	128,541	10.35	13,301	149,311	133,903	7.91	10,587	159,775
1998	10.6	3.03	144,850	12,308	146,622	149,311	10.67	15,927	147,554	159,775	7.91	12,636	162,770
1999	11.0	2.58	146,622	12,308	149,052	147,554	10.97	16,186	145,782	162,770	8.03	13,064	166,132
2000	-4.1	2.26	149,052	12,308	131,198	145,782	11.42	16,649	123,896	166,132	8.28	13,753	146,199
2001	1.8	2.68	131,198	12,308	121,034	123,896	11.70	14,492	111,376	146,199	8.99	13,136	135,462
2002	-4.4	3.28	121,034	12,308	103,967	111,376	12.17	13,555	93,539	135,462	9.84	13,326	116,789
2003	20.9	4.33	103,967	12,308	110,861	93,539	12.63	11,818	98,841	116,789	10.98	12,821	125,749
2004	1.8	3.76	110,861	12,308	100,296	98,841	13.37	13,217	87,137	125,749	11.53	14,498	113,217
2005	-0.1	3.68	100,296	12,308	87,916	87,137	14.16	12,338	74,738	113,217	12.54	14,195	98,940
2006	9.0	3.78	87,916	12,308	82,437	74,738	15.11	11,290	69,179	98,940	13.78	13,638	93,007
2007	1.1	3.67	82,437	12,308	70,931	69,179	16.36	11,317	58,524	93,007	15.09	14,032	79,878
2008	1.3	3.85	70,931	12,308	59,367	58,524	18.08	10,579	48,554	79,878	16.84	13,448	67,274
2009	18.2	6.51	59,367	12,308	55,612	48,554	20.20	9,806	45,789	67,274	20.01	13,464	63,588
2010	8.0	4.92	55,612	12,308	46,748	45,789	23.40	10,714	37,865	63,588	22.66	14,410	53,089
2011	-6.1	4.47	46,748	12,308	32,332	37,865	28.12	10,646	25,552	53,089	27.33	14,510	36,217
2012	9.6	4.87	32,332	12,308	21,938	25,552	36.02	9,203	17,912	36,217	35.47	12,847	25,605
2013	27.8	4.71	21,938	12,308	12,308	17,912	51.98	9,311	10,991	25,605	51.51	13,188	15,868
2014	15.0	4.02	12,308	12,308	0	10,991	100.00	10,991	0	15,868	100.00	15,868	0

Table 5
20-Year Decumulation Starting in 1973, based on buy-and-hold S&P 500 real returns

In this Table we present statistics for an investor beginning a 20-year decumulation period beginning in 1973, where investment returns are all driven from a buy-and-hold investment in the S&P500. The second column in the table presents the annual, real return achieved from investing in a buy-and-hold S&P 500 equity portfolio. The third column in the table presents the 1/CAPE value (EY) at the start of each decumulation year. The columns under the heading "Perfect Foresight PWA", present the value of the investment fund at the start of each year, the annual, perfect foresight withdrawal amount, and the value of the investment fund at the end of each year respectively. The columns under the heading "Monte Carlo Median PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the Median PWA has been determined by the Monte Carlo technique described in the text which is applied using data up to the start of the next withdrawal year. The columns under the heading "CAPE-Based PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the PWA has been determined at the start of each year by the CAPE regression described in the text.

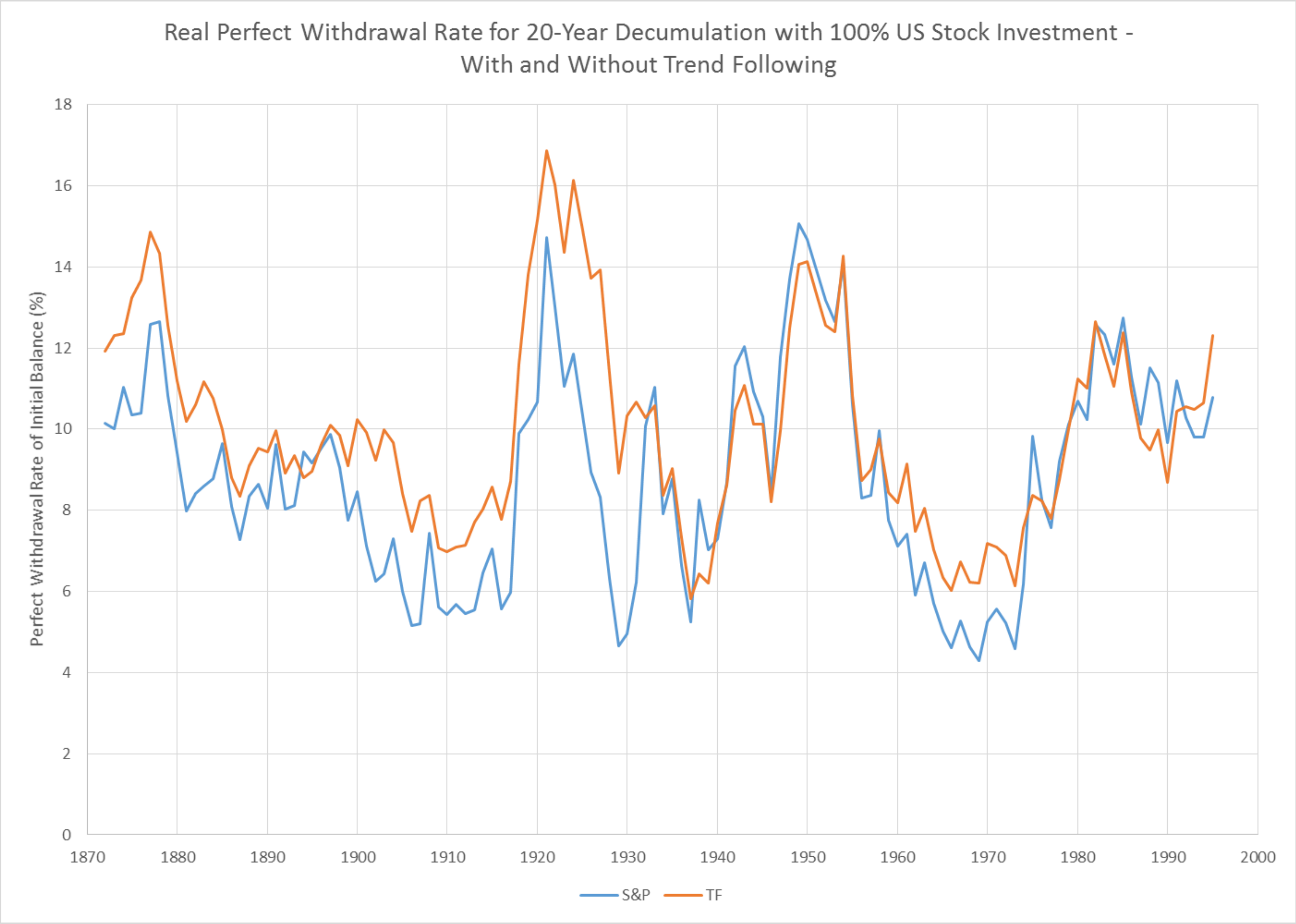
			Perfect Foresight PWA			Monte Carlo Median PWA				CAPE-Based PWA			
Year	Real Ret (%)	EY Start	Start ($)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
1973	-23.5	5.36	100,000	4,591	72,972	100,000	8.84	8,844	69,719	100,000	7.48	7,477	70,765
1974	-34.2	7.41	72,972	4,591	44,987	69,719	8.86	6,178	41,802	70,765	8.79	6,222	42,461
1975	29.1	12.06	44,987	4,591	52,147	41,802	8.87	3,707	49,177	42,461	11.39	4,837	48,568
1976	16.8	9.76	52,147	4,591	55,563	49,177	9.20	4,523	52,172	48,568	10.56	5,130	50,752
1977	-12.2	8.62	55,563	4,591	44,759	52,172	9.62	5,019	41,405	50,752	10.33	5,245	39,960
1978	-1.1	10.33	44,759	4,591	39,728	41,405	9.87	4,089	36,907	39,960	11.57	4,623	34,949
1979	4.3	11.10	39,728	4,591	36,646	36,907	10.25	3,782	34,548	34,949	12.41	4,336	31,928
1980	15.7	11.43	36,646	4,591	37,102	34,548	10.76	3,717	35,684	31,928	13.11	4,185	32,110
1981	-10.5	10.65	37,102	4,591	29,102	35,684	11.42	4,076	28,293	32,110	13.31	4,273	24,918
1982	14.8	12.77	29,102	4,591	28,142	28,293	12.09	3,422	28,555	24,918	15.24	3,796	24,250
1983	18.7	11.81	28,142	4,591	27,948	28,555	13.02	3,719	29,474	24,250	15.56	3,774	24,299
1984	0.7	10.19	27,948	4,591	23,532	29,474	14.14	4,168	25,493	24,299	15.57	3,783	20,669
1985	26.6	10.42	23,532	4,591	23,970	25,493	15.45	3,939	27,278	20,669	16.92	3,496	21,732
1986	22.8	8.55	23,970	4,591	23,793	27,278	17.21	4,695	27,725	21,732	17.48	3,799	22,016
1987	-4.3	7.10	23,793	4,591	18,367	27,725	19.62	5,438	21,318	22,016	18.93	4,167	17,073
1988	13.8	7.47	18,367	4,591	15,674	21,318	22.82	4,864	18,718	17,073	22.33	3,813	15,085
1989	24.4	6.80	15,674	4,591	13,790	18,718	27.72	5,189	16,833	15,085	26.85	4,050	13,731
1990	-8.0	5.67	13,790	4,591	8,466	16,833	35.86	6,036	9,937	13,731	34.37	4,719	8,293
1991	18.4	6.31	8,466	4,591	4,591	9,937	51.87	5,154	5,664	8,293	51.01	4,231	4,812
1992	12.2	5.42	4,591	4,591	0	5,664	100.00	5,664	0	4,812	100.00	4,812	0

Table 6
20-Year Decumulation Starting in 1973, based on real S&P 500 returns with trend following overlay

In this Table we present statistics for an investor beginning a 20-year decumulation period beginning in 1973, where investment returns are all driven by the real return on the S&P500 with a trend following overlay. The second column in the table presents the annual, real return achieved from investing in a buy-and-hold S&P 500 equity portfolio. The third column in the table presents the 1/CAPE value (EY) at the start of each decumulation year. The columns under the heading "Perfect Foresight PWA", present the value of the investment fund at the start of each year, the annual, perfect foresight withdrawal amount, and the value of the investment fund at the end of each year respectively. The columns under the heading "Monte Carlo Median PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the Median PWA has been determined by the Monte Carlo technique described in the text which is applied using data up to the start of the next withdrawal year. The columns under the heading "CAPE-Based PWA" present: the value of the investment fund at the start of each year; the annual, perfect foresight withdrawal amount as a proportion of the fund; the cash withdrawal amount; and the value of the investment fund at the end of each year respectively, where the PWA has been determined at the start of each year by the CAPE regression described in the text.

			Perfect Foresight PWA			Monte Carlo Median PWA				CAPE-Based PWA			
Year	Real Ret (%)	EY Start	Start ($)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)	Start ($)	Withdrawal (%)	Withdrawal ($)	End ($)
1973	-15.3	5.36	100,000	6,148	79,522	100,000	10.20	10,203	76,086	100,000	8.81	8,806	77,270
1974	-4.0	7.41	79,522	6,148	70,429	76,086	10.25	7,800	65,545	77,270	10.13	7,830	66,652
1975	8.3	12.06	70,429	6,148	69,592	65,545	10.39	6,810	63,587	66,652	12.80	8,535	62,918
1976	13.0	9.76	69,592	6,148	71,671	63,587	10.63	6,757	64,199	62,918	11.86	7,462	62,648
1977	-4.9	8.62	71,671	6,148	62,304	64,199	10.96	7,036	54,354	62,648	11.58	7,258	52,669
1978	-5.5	10.33	62,304	6,148	53,075	54,354	11.20	6,087	45,619	52,669	12.78	6,731	43,417
1979	-2.4	11.10	53,075	6,148	45,820	45,619	11.54	5,266	39,402	43,417	13.53	5,874	36,658
1980	13.4	11.43	45,820	6,148	44,994	39,402	11.96	4,713	39,341	36,658	14.13	5,180	35,699
1981	-3.9	10.65	44,994	6,148	37,328	39,341	12.49	4,913	33,083	35,699	14.27	5,096	29,408
1982	20.5	12.77	37,328	6,148	37,579	33,083	13.17	4,357	34,621	29,408	15.92	4,681	29,801
1983	18.7	11.81	37,579	6,148	37,300	34,621	14.02	4,854	35,325	29,801	16.22	4,835	29,627
1984	-1.3	10.19	37,300	6,148	30,760	35,325	15.10	5,334	29,614	29,627	16.29	4,825	24,491
1985	26.6	10.42	30,760	6,148	31,147	29,614	16.32	4,832	31,362	24,491	17.51	4,288	25,567
1986	22.8	8.55	31,147	6,148	30,692	31,362	18.07	5,667	31,546	25,567	18.29	4,677	25,646
1987	11.6	7.10	30,692	6,148	27,386	31,546	20.28	6,399	28,059	25,646	19.97	5,123	22,899
1988	2.5	7.47	27,386	6,148	21,764	28,059	23.49	6,590	22,000	22,899	23.27	5,329	18,006
1989	24.4	6.80	21,764	6,148	19,430	22,000	28.11	6,185	19,677	18,006	27.84	5,012	16,167
1990	-10.8	5.67	19,430	6,148	11,845	19,677	36.16	7,114	11,203	16,167	35.49	5,738	9,301
1991	7.9	6.31	11,845	6,148	6,148	11,203	51.99	5,824	5,805	9,301	51.75	4,813	4,842
1992	9.5	5.42	6,148	6,148	0	5,805	100.00	5,805	0	4,842	100.00	4,842	0

Figure 1



Real Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock Investment - With and Without Trend Following
Perfect Withdrawal Rate of Initial Balance (%)
18
16
14
12
10
8
6
4
2
0
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
S&P
TF

Figure 2

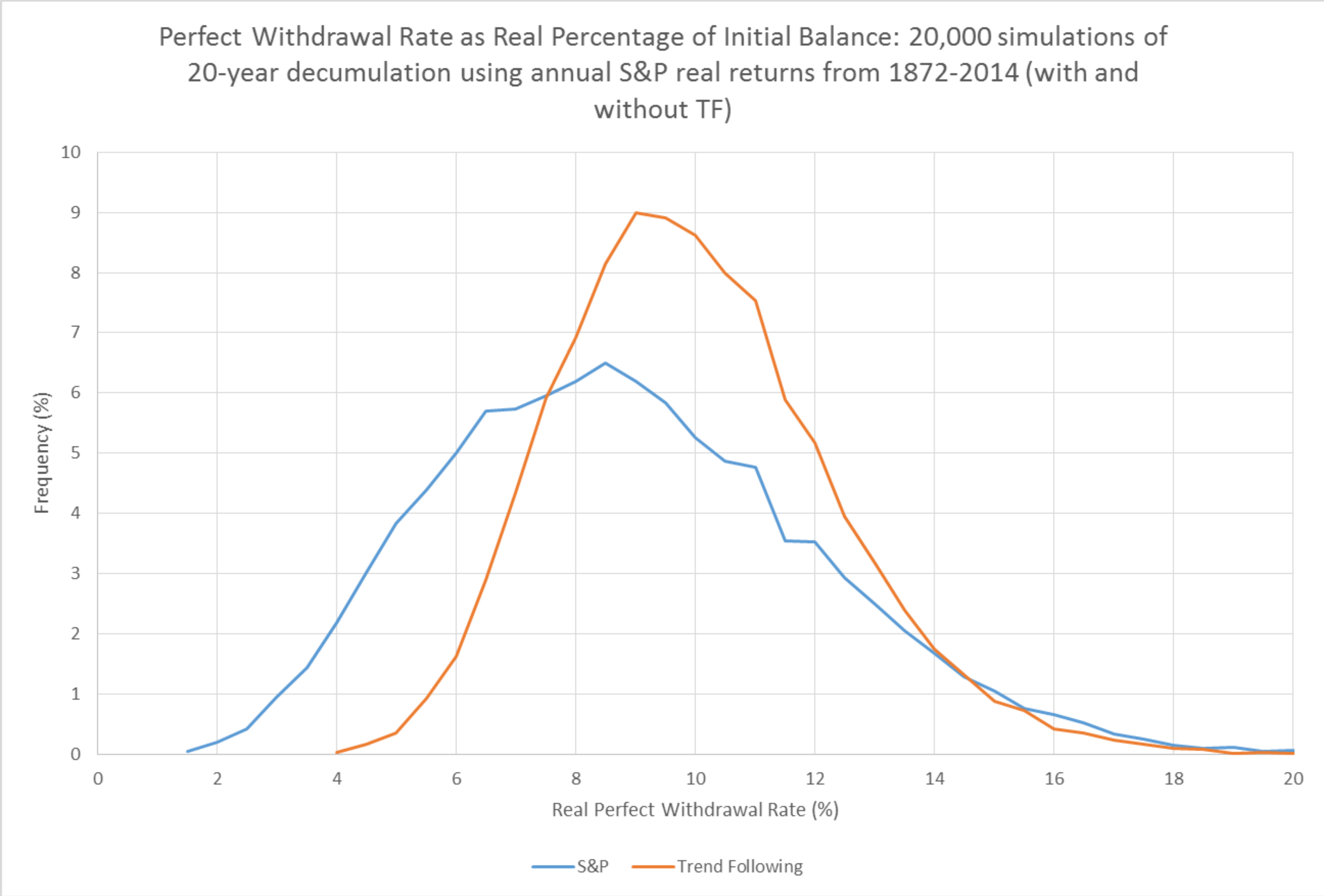
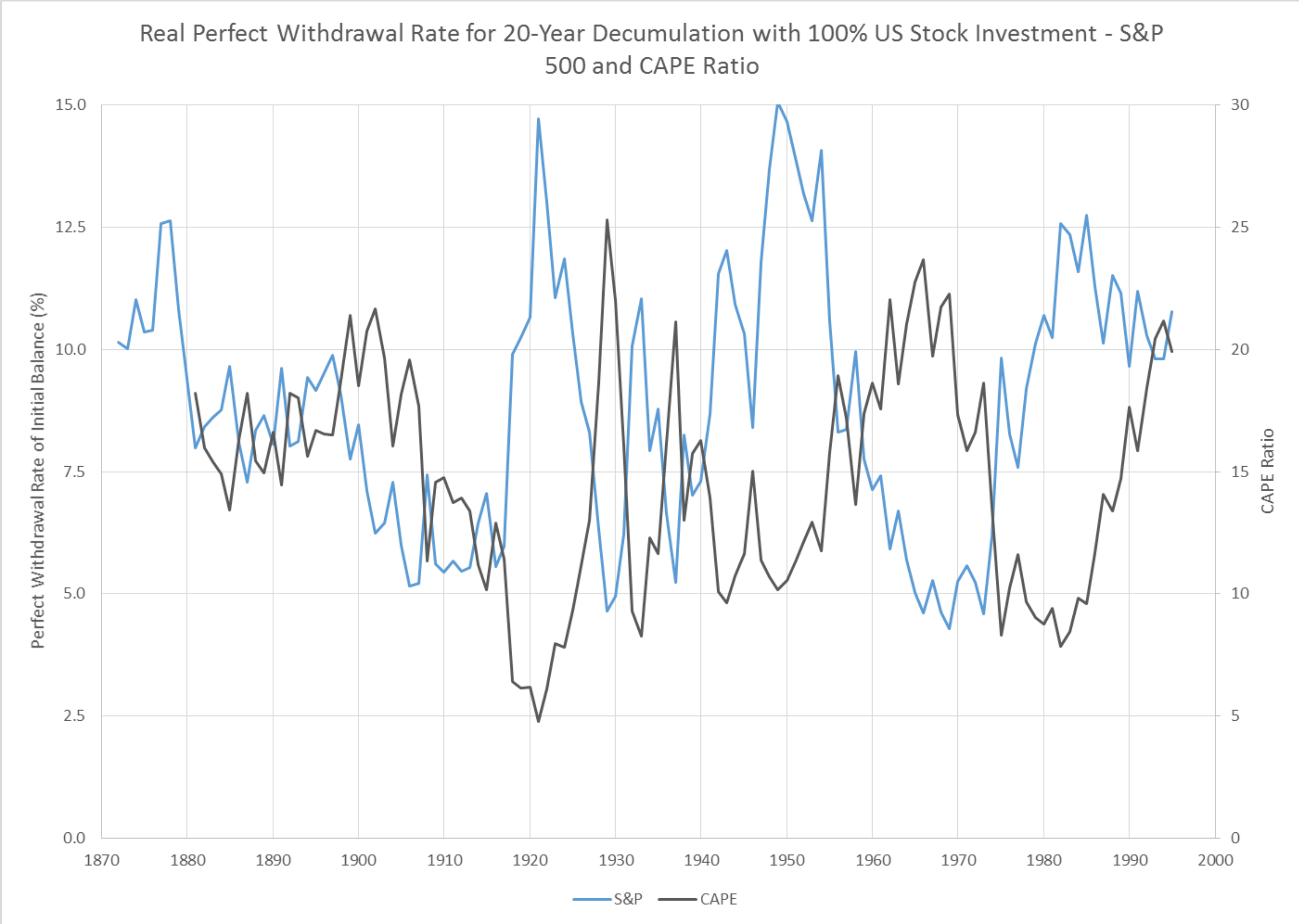

Perfect Withdrawal Rate as Real Percentage of Initial Balance: 20,000 simulations of 20-year decumulation using annual S&P real returns from 1872-2014 (with and without TF)
Frequency (%)
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12
14
16
18
20
Real Perfect Withdrawal Rate (%)
S&P
Trend Following

Figure 3


Real Perfect Withdrawal Rate for 20-Year Decumulation with 100% US Stock Investment - S&P 500 and CAPE Ratio
Perfect Withdrawal Rate of Initial Balance (%)
CAPE Ratio
15.0
12.5
10.0
7.5
5.0
2.5
0.0
30
25
20
15
10
5
0
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
S&P
CAPE